The Structure of the Higher Education System, Productivity, and Inequality

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Abstract

In this paper, I introduce the higher education system into a growth model and analyze the bilateral feedback between socioeconomic composition and the structure of the higher education system and the effects of the structure on productivity and inequality. The results of this paper indicate that a country with a larger population, a larger proportion of high-ability agents, and a higher return to high-quality education, tends to have a diversified system, which consists of a mix of institutions that differ in prestige, quality, and selectivity of students. Otherwise, the country tends to have a unified system, which has very small variance in the quality of universities. Compared with the unified system, the diversified system adds more productivity on high-ability agents and reduces the number of uneducated high-ability agents, thereby increasing the aggregate productivity. Furthermore, even though the diversified system increases the Gini coefficient, it is, nevertheless, a Pareto improvement. Moreover, transforming into a diversified higher education system could eliminate poverty trap and reduce inequality in the long-run.

I. Introduction

The structure of the higher education system differs across countries. In some countries, Germany for example, higher education is provided by institutions with very small variance in the quality. This is referred to as the "unified" system. In some other countries, however, the U.S. for example, the higher education system consists of a mix of institutions that differ in prestige, quality, and selectivity of students, and is referred to as the "diversified" system. On the other hand, some countries implement policies that could reshape the structure of the higher education system. For example, China has proposed the 985 Project, which is aimed to promote a selected group of Chinese universities to the world first-rate quality (Zhang, Patton and Kenney (2013)). In this paper, I analyze how market size determines the structure of the higher education system. Moreover, the structure of the higher education system affects the investment in human capital, thus the income. Therefore, I also analyze the effects of the structure on productivity and inequality, especially the effects on the dynamics of inequality in the long-run.

The results of this paper show that the size of market is the major reason for the difference in the structure of the higher education system. Compared with Germany, the higher education in the U.S. has a larger market, for the reason that the U.S. population is larger and English is more wildly used than German. Therefore, the higher education system in the U.S. is more diversified. Figure 1 shows the distribution of American and German universities among the top 900 universities in the world (U.S. News (2016)), with the total number of the universities of each country normalized to one. We can find that, compared with the distribution of American universities, which is illustrated by the dashed line, the distribution of German universities, illustrated by the solid line, is more concentrated. Moreover, the U.S. has a clear advantage in the top 100 universities.

Furthermore, the analysis in this paper argues that because the diversified system adds more human capital on high-ability workers, and also attracts more high-ability people to invest in education, it increases productivity but also increases inequality. This result is

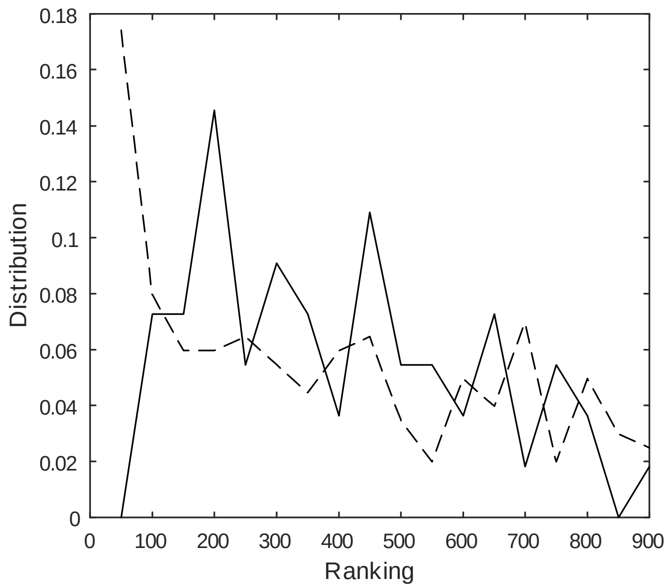

Figure 1: The Distribution of Universities in Germany and the U.S.

consistent with the data shown in Table 1, which states that the U.S. has higher college attainment rate and fewer under-matched students, the students who are smart enough to enter universities but have not. Moreover, the U.S. has higher productivity and higher Gini coefficient. However, this paper also derive a different prediction that transforming to the diversified system could eliminate poverty trap and reduce inequality in the long-run, since it offers poor people opportunities to invest in education and earn higher wage when they are high-ability.

Table 1. Productivity and Inequality

Country	U.S.	Germany
College Attainment	11.7	11.2
Under-match	0.055	0.104
Labour productivity	64.1	58.3
Gini (before taxes and transfers)	0.513	0.508
Gini (post taxes and transfers)	0.396	0.292

Source: Cooper and Liu (2016) and OECD Stat (2012, 2013)

This paper analyzes an OLG model in a small open economy, which consists of agents with heterogeneous ability: high or low. Agents supply their labour as educated or uneducated workers. Because of credit market imperfection, individuals can only borrow a limited amount in order to finance their investment in education, as in other inequality and growth models (Galor and Zeira (1993); Banerjee and Newman (1993)). Education investment thus depends on the distribution of endowments, which in turn, depends on past wages. In the higher education system, there are two universities and two possible tiers in which the universities could operate. If a university operates in the low tier, it provides low-quality education to both low-ability and high-ability agents. If it operates in the high tier, it provides high-quality education that is only available for high-ability agents. The cost of operating in a tier is fixed but it costs more to operate in the high tier. The human capital that an agent can obtain depends on both her ability and the quality of her education. The wages for different workers are constant and depend on their human capital.

In each period, agents decide on their education and become educated or uneducated workers. The two universities maximize their profit by deciding on the tier to operate in. Given the distribution of wealth, the equilibrium is an allocation of workers, their education choices, and universities' choices of operating tiers. The analysis of the Nash equilibrium generates a first result that an economy with a larger population, a larger proportion of high-ability agents, and a higher return to high-tier education, tends to have a diversified system, which means that one university operates in each tier. The reason is that, for high-ability agents, compared with the low-tier education, high-tier education brings higher wage, thus attracting more students. However, since operating in the high tier also generates higher fixed cost, it is not profitable to specialize in providing high-tier education for high-ability agents, unless the potential market for high-tier education is large enough.

The diversified system adds more human capital on high-ability agents, and brings a higher wage to them, thus attracting more high-ability agents to invest in education and reducing the number of uneducated workers. As a result, the aggregate human capital and productivity is increased. Since the income for educated high-ability workers is higher under the diversified system, the inequality is increased. However, since the wages for uneducated and educated low-ability workers are not changed, transforming from the unified system to the diversified one is a Pareto improvement.

In the long-run, wages affect the wealth and bequests of agents. Therefore, the distribution of wealth becomes endogenous as well. To determine the long-run dynamics of the model, I solve for a steady state which can only be one of two different cases. In the first type of steady state, the egalitarian steady state, the wealth of every individual converges to the similar levels, and all the agents are educated or uneducated. In this case, the only difference in income is caused by the difference in ability. In the other type of steady state, the polarized steady state, there are two different long-run wealth levels: the rich group of educated workers, and the poor group of uneducated workers. The agents of the poor group can never afford education, no matter they are high-ability or low-ability. Which steady state is attained is determined by the real wages for different types of workers. Transforming from the unified system to a diversified system could increase the wage for educated high-ability agents, which makes education profitable for poor agents, if they are high-ability. Therefore, in each generation, a fraction of poor agents can flee the poverty trap by investing in education. Eventually, all the agents will be educated, and the only difference in income is caused by the difference in ability.

This paper resembles the work of Galor and Zeria (1993) on the credit market imperfection and the dynamic distribution of wealth. However, by introducing heterogeneous ability and endogenous higher education system, the model in this paper generates more dynamics which lead to various steady states.

Very little economic literature addresses the determinants of the structure of the higher education. There are some illuminating education literature on this issue, for example, Fairweather (2000) and Teichler (2008). Market size has been used as explanation for industrialization in growth literature, for example, Murphy, Shleifer and Vishny (1989a) and

Murphy, Shleifer and Vishny (1989b). Particularly, Shaked and Sutton (1987) and Melitz and Ottaviano(2008) analyze the relationship between market size and industry structure.

This paper also complements the literature on the effects of the structure of the higher education. The paper most related to this paper is Allmendinger (1989), who categorizes education systems into standardized ones and stratified ones, and analyzes their effects on labour market. More generally, Krueger and kumar (2004) use the difference in the characteristics of education in the U.S. and Germany to explain the growth differences between the two countries. Also, Shavit ed. (2007) provide vast empirical studies on the effect of the structure of the higher education on inequality.

The remainder of this paper is organized as follows: in Section II, I set up the basic model. Section III examines the equilibrium of the higher education system and its effects. Section IV discusses the dynamic version of the model and its steady states. Section V offers some concluding remarks.

II. The Model

We consider a small open economy. The economy is populated by a continuum of agents with a mass of L, with heterogeneous ability b. An agent a has low ability if $b_i = 1$ and she has high ability if $b_i = 2$. The proportion of the agents with low ability is λ while the proportion of the agents with high ability is $1 - \lambda$.

There is a continuum of competitive firms, which hire different types of workers to produce a single good. In this small open economy, the price of the good is equal to the world market price, which is exogenous and normalized to 1. Agents can save and borrow to finance their education. Both the saving and borrowing world market interest rates are also exogenous and constant over time.

A. Imperfect Credit Market

The credit market is imperfect, in that there is a spread between the risk free saving and borrowing interest rates denoted by r and b, respectively. Set the spread be denoted by $\beta \geq 0$, so that $1 + i = \beta(1 + r)$. This assumption is borrowed from the one of Galor and Zeira (1993), which is a tried and tested way to incorporate borrowing constrains. Borrowing constraints are more severe for households than for firms. For simplicity I assume that firms can borrow at rate r.

B. Higher Education System

There are two universities, $u = (v, w)$, operating in the higher education system. There are two possible tiers of education, $e = (1, 2)$, from which each university could choose to provide. If $e_u = 1$, university u chooses to provide low tier education, which is available for agents with both low and high ability. If $e_u = 2$, university u chooses to provide high tier education, which is only available for the agents with high ability. Providing different tiers of education generates different fixed cost κ_{e_u} for each university, and it costs more to provide high tier education than to provide low tier education: $\kappa_2 > \kappa_1$. The number of enrolled students university u could attract, N_u, depends on its own choice of which tier education to provide, as well as the choice of the other university. Therefore, the profit one university can make by providing education, y_u, also depends on both e_w and e_v:

$$y_u(e_w, e_v) = \tau \cdot N_u - \kappa_{e_u} \tag{1}$$

where τ is the tuition fee, and it costs the same for a agent to take high tier or low tier education. The problem of each university is:

(2) $$\underset{e_u}{Max} \quad y_u(e_w, e_v)$$

According to the choices of two universities, there are three possible scenarios in the higher education system: both universities provide low tier education; one provides low tier education and the other provides high tier education; both provide high tier education.

Meanwhile, the higher education adds human capital on agents. The amount of human capital Z that an agent can obtain by taking education is determined by both her ability b and the tier of education e that she has taken. The human capital of uneducated workers is normalized to 1. Moreover, the human capital $Z_{b,e}$ of agents with different ability and education follows the following order:

(3) $$1 < Z_{1,1} < Z_{2,1} < Z_{2,2}$$

Wage of agents depend on their human capital. The wage of educated workers is $w_{b,e} = A_s Z_{b,e}$, where A_s governs the productivity that is not related to human capital. The wage of uneducated workers is $w_u = A_u$. According to Equation (3), the wages of workers with different levels of ability and education follow:

(4) $$w_u < w_{1,1} < w_{2,1} < w_{2,2}$$

C. Households

Each agent lives for two periods in overlapping generations: young and old. In period t, a young agent receives bequest $x_{a,t}$ from her parent and decides on her education: she has the choice either to invest in education or not. The cost of investing in human capital is τ.

When old, agents work as educated or uneducated workers, depending on their education level, and earn educated or uneducated wages $w_{b,e,t}$ or $w_{u,t}$. Agents only consume and leave bequests to children in the second period of their life. This framework closely follows Galor and Zeira (1993). Agent a receives lifetime utility $u_{a,t}$ from both consumption $c_{a,t}$ and the bequest $x_{a,t}$:

$$u_{a,t} = \theta \ln c_{a,t} + (1-\theta) \ln x_{a,t} \tag{5}$$

The parameter $\theta \in (0,1)$ determines the saving rate. The optimal choice of $c_{a,t}$ and $x_{a,t}$ for an agent a in the second period of his life maximizes $u_{a,t}(c_{a,t}, x_{a,t})$ subject to the budget constrain:

$$c_{a,t} + x_{a,t} = \pi_{a,t} \tag{6}$$

where $\pi_{a,t}$ is lifetime income. Therefore the individual utility only depends on lifetime income $\pi_{a,t}$ and is strictly increasing in the income. As a result, agent's problem is to maximize lifetime income by deciding on her education. An agent working as uneducated without investing gets income:

$$\pi_{a,t} = w_{u,t} + x_{a,t-1}(1+r)$$

An agent with bequest $x_{a,t} \geq \tau$, who invests in higher education, gets:

$$\pi_{a,t} = w_{b,e,t} + (x_{a,t-1} - \tau)(1+r)$$

An agent, who receives bequest $x_{a,t} < \tau$ and invests, needs to borrow and gets income:

$$\pi_{a,t} = w_{b,e,t} - (\tau - x_{a,t-1})(1+i)$$

As discussed in the previous section, there are three possible scenarios in the higher education system. I look into the supply of different types of workers, given the structure of the higher education system.

If two universities operate in Tier 1, an agent with $b = 1$, or a low-ability agent is indifferent between investing and not if bequest $x_{a,t-1}$, is equal to:

$$f_{1,1} = \frac{1}{i-r}[w_u(1+r) + \tau(1+i) - w_{1,1}] \tag{7}$$

An agent with $b = 2$, or a high-ability agent is indifferent between investing and not if bequest $x_{a,t}$, is equal to:

$$f_{2,1} = \frac{1}{i-r}[w_u(1+r) + \tau(1+i) - w_{2,1}] \tag{8}$$

Therefore, given wages, w_u and $w_{1,1}$, all low-ability agents with endowment greater than $f_{1,1}$ will invest in human capital and agents with endowment smaller than $f_{1,1}$ will not invest. If $F(\cdot)$ is the CDF of the distribution of initial be quest, then the supply of educated low-ability workers is:

$$L_{1,1,t}\,(w_{1,1}, w_u) = \lambda L[1 - F(f_{1,1})] \tag{9}$$

Similarly, the supply of educated high-ability workers is:

$$L_{2,1,t}\,(w_{2,1}, w_u) = (1-\lambda)L[1 - F(f_{2,1})] \tag{10}$$

And the supply of uneducated workers is

$$L_{u,t} = L - L_{1,1,t} - L_{2,1,t}$$

If two universities provide different types of education, the low-ability agents can only take low tier education, and high-ability agents will only choose high tier education, if they can afford that, because education of higher tier brings higher wage and costs the same. An agent with $b = 1$ is indifferent between investing and not if bequest $x_{a,t-1}$, is equal to:

$$f_{1,1} = \frac{1}{i-r}[w_u(1+r) + \tau(1+i) - w_{1,1}] \tag{11}$$

An agent with $b = 2$ is indifferent between investing and not if bequest $x_{a,t}$, is equal to:

$$f_{2,2} = \frac{1}{i-r}[w_u(1+r) + \tau(1+i) - w_{2,2}] \tag{12}$$

Therefore, given wages, $w_{u,,t}$ and $w_{1,1,t}$, all low-ability agents with endowment greater than $f_{1,1,t}$ will invest in human capital and agents with endowment smaller than $f_{1,1,t}$ will not invest. Then the supply of educated low-ability workers is:

$$L_{1,1,t}\,(w_{1,1,}, w_u) = \lambda L[1 - F(f_{1,1})] \tag{13}$$

Similarly, the supply of educated high-ability workers is:

$$L_{2,2,t}\,(w_{2,2}, w_u) = (1-\lambda)L[1 - F(f_{2,2})] \tag{14}$$

And the supply of uneducated workers is

$$L_{u,t} = L - L_{1,1,t} - L_{2,2,t}$$

Since the wage for high-ability workers with high-tier education $w_{2,2}$ is higher than the wage for them with low-tier education $w_{2,1}$, the threshold for them to invest follows: $f_{2,2} < f_{2,1}$. As a result, $L_{2,2,t}$, the supply of educated high-ability workers when two universities provide different types of education is larger then $L_{2,1,t}$, the supply of educated high-ability workers when two universities operate in Tier 1.

If two universities operate in Tier 2, the agents with $b = 1$ cannot invest in education. An agent with $b = 2$ is indifferent between investing and not if bequest $x_{a,t-1}$, is equal to:

$$f_{2,2} = \frac{1}{i-r}[w_u(1+r) + \tau(1+i) - w_{2,2}] \tag{15}$$

The supply of educated high-ability workers is:

$$L_{2,2,t}\left(w_{2,2}, w_u\right) = (1-\lambda)L[1 - F(f_{2,2})] \tag{16}$$

And the supply of uneducated workers is

$$L_{u,t} = L - L_{2,2,t}$$

III. The Equilibrium of the Higher Education System

In this section, I look at the Nash equilibrium of the higher education system and its effects on productivity and inequality. Given ability, initial wealth and returns to different tiers of education, agents make decision on education. Anticipating the number of enrolled students, universities make decision on tiers to operate in. Formally,

Definition 1. *A Nash equilibrium of the higher education system is a pair of strategies* $\{e_w, e_v\}$*, that maximize each university's profit* y_u*, given the other university's choice.*

Proposition 2. *There always exists a unique Nash equilibrium in the higher education system and there are three possible scenarios : the Nash equilibrium is that both two universities operate in Tier 1, which means that* $\{e_w = 1, e_v = 1\}$*; the Nash equilibrium is that one university operates in each tier, which means that* $\{e_w = 1, e_v = 2\}$ *or* $\{e_w = 2, e_v = 1\}$ *; the Nash equilibrium is that both two universities operate in Tier 2, which means that* $\{e_w = 2, e_v = 2\}$ *.*

The proof is in the appendix. Since the scenario that both universities give up low ability market and only operate in the high tier is rare in the real world, this paper focuses on the first two scenarios: the unified system where both universities provide Tier 1 education to all agents, and the diversified system where one university only provides Tier 1 education to low-ability agents and the other university only provide Tier 2 education to the high-ability agents.

Proposition 3. *(i) Compared with the unified higher education system, the diversified system exists when the proportion of high-ability agents* $1 - \lambda$ *is larger;*

(ii) There exists a $\overset{\circ}{\lambda}$*, when* $1 - \lambda > 1 - \overset{\circ}{\lambda}$*, compared with the unified higher education system, the diversified system exists when the population* L *is larger.*

(iii) Compared with the unified higher education system, the diversified system exists when $Z_{2,2}$*, the human capital that Tier 2 education can add on high-ability agents is higher.*

Proposition 3 (i) and (ii) state the effects of market size on the higher education system. Because the existence of the fixed cost for a university to provide education, it is not profitable for university to operate in Tier 2 unless high-ability population, i.e. the number of the potential buyers of the Tier 2 education, is large enough. This could explain that the higher education systems in the English-speaking countries, the UK and the US for example, is more diversified.

Proposition 3 (iii) states that if the return to high-ability agents with better education is higher, more high-ability agents are willing to invest in education. Thus, it is profitable to provide education specially for high-ability agents.

A. The Effect of Higher Education System on Productivity

The productivity of an economy is affected by the aggregate human capital obtained by workers through taking education. Since the population and technology is constant, the productivity is determined by the average human capital $\overline{z}_t$, which can be measured as:

$$\begin{aligned} \overline{z}_t &= (\sum_{e=1}^{2}\sum_{b=1}^{2} Z_{b,e}L_{b,e,t} + \sum_{b=1}^{2} L_{u,t})/L \\ &= (Z_{1,1}L_{1,1,t} + Z_{2,1}L_{2,e,t} + \sum_{b=1}^{2} L_{u,t})/L \end{aligned} \tag{17}$$

By comparing the average human capital with unified and diversified higher education system, we can have the following proposition. The proof is in the appendix.

Proposition 4. *Compared with the unified higher education system, the diversified system brings higher average human capital, thus increase the productivity.*

The diversified higher education system could increase the average human capital, because it could reduce two types of mismatch. The first type is the educated-uneducated mismatch. Because of the credit market imperfection, a fraction of high-ability agents are constrained by the endowment and do not invest in education. Tier 2 education adds more human capital on high-ability agents than Tier 1 education does, thus the wage of high-ability agents with Tier 2 education is higher than the wage of high-ability agents with Tier 1 education. Therefore, the diversified higher education system could attract more high-ability agents to invest in education. The second type of mismatch is the mismatch between the types of education and ability. Compared with the unified higher education system, the diversified system could support high-ability agents overproportionally and add on more human capital on them. By reducing these two types of mismatch, the diversified higher education system increases productivity of an economy.

B. The Effect of Higher Education System on Inequality

Taking the initial distribution of endowments as given, the distribution of income is determined by the wages of different types of workers, which is affected by the structure of higher education. To analyze inequality in an economy, let us consider the following assumption:

Assumption: *At the beginning of period t, bequests follow a uniform distribution on $[0, M]$, with $M > \tau$. The distribution of bequest is independent from the one of ability.*

From the above section, we know that all the agents with the same ability can be divided into three groups, according to their investment decisions: agents will not invest if their bequest is lower than the threshold; agents will invest and borrow if their endowments are higher than the threshold but lower than the cost of education; agents will invest in education without borrowing, if they receive bequests higher than the cost of education. Bequests of agents with same ability evolve as follows:

$$x_{a,t} = \begin{cases} (1-\theta)[(w_u + x_{a,t-1})(1+r)], if\, x_{a,t-1} < f_{b,e} \\ (1-\theta)[w_{b,e} - (\tau - x_{a,t-1})(1+i)], if\, f_{b,e} \leqslant x_{a,t-1} < \tau \\ (1-\theta)[w_{b,e} + (x_{a,t-1} - \tau)(1+r)], if\, x_{a,t-1} \geqslant \tau \end{cases} \tag{18}$$

I suppose that $(1-\theta)(1+r) < 1$ to focus on interesting dynamics, following Galor and Zeira (1993). This assumption rules out the possibility that the incomes of agents in poor group converge to zero in the long-run or the incomes of agents in rich group diverge. Since the distribution of bequest is independent from the one of ability, an agent will be low-ability with the probability of λ. Therefore, given the endowments, the expected bequests that agents will leave to the next generation are:

$$
(19)\qquad x_{a,t} = \begin{cases} (1-\theta)[(w_u + x_{a,t-1})(1+r)], & if\, x_{a,t-1} < f_{2,1} \\ \\ \lambda(1-\theta)[(w_u + x_{a,t-1})(1+r)] & \\ \quad + (1-\lambda)(1-\theta)[w_{2,e} - (\tau - x_{a,t-1})(1+i)], & if\, f_{2,1} \leqslant x_{a,t-1} < f_{1,1} \\ \\ \lambda(1-\theta)[w_{1,1} - (\tau - x_{a,t-1})(1+i)] & \\ \quad + (1-\lambda)(1-\theta)[w_{2,e} - (\tau - x_{a,t-1})(1+i)], & if\, f_{1,1} \leqslant x_{a,t-1} < \tau \\ \\ \lambda(1-\theta)[w_{1,e} + (x_{a,t-1} - \tau)(1+r)] & \\ \quad + (1-\lambda)(1-\theta)[w_{2,e} + (x_{a,t-1} - \tau)(1+r)], & if\, x_{a,t-1} \geqslant \tau \end{cases}
$$

Transforming from a unified system to a diversified one not only increases the total bequest but also affects the share of each group. With equation (19), we can calculate for the total bequest of the economy, and analyze the share of the bequest of each group. The following proposition states the effect of the structure of the higher education system on inequality of an economy. The proof is in the appendix.

Proposition 5. *Compared with the unified higher education system, the diversified system increases the Gini coefficient, but it is a Pareto improvement.*

The proposition states that, compared with the unified higher education system, the diversified system increases inequality. It increases the share of the bequest of relatively rich groups, since it increases the wage of educated high-ability agents. However, it does not affect the income of educated low-ability agents and uneducated agents. Therefore, this change is a Pareto improvement.

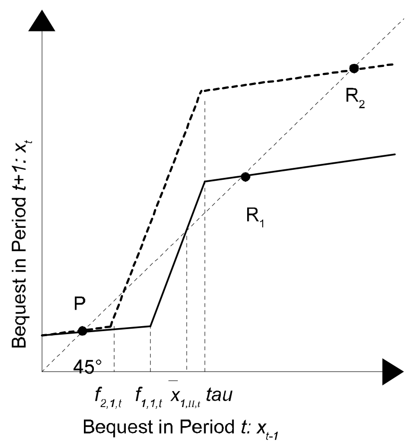

Figure 2: Transition of the Distribution of Wealth

IV. The Dynamic Model

In this section, I develop the model to a dynamic version, by taking into account the transition of the distribution of wealth. Wages determine the income of each agent and then the bequest she gives to her child. As a result, in the long-run, the distribution of wealth becomes endogenous as well. To examine the long-run evolution of the economy and its skill premium, I firstly characterize its steady state, and then analyze how the structure of the higher education system affect the steady state, by affecting the transition. For the reason that credit market imperfection is an important reason of inequality, I only look at the dynamics with $(1-\theta)(1+i) > 1$, which means the scenario that credit market imperfection is severe.

The transition of the distribution of wealth is determined by the real wages of different workers. Therefore, there exist many possible transitions, which lead to different types of steady states. Figure 2 illustrates an example of transitions, assuming that the higher education system is unified. This figure shows the bequests $x_{a,t}$ that agents will leave to next generation, given their endowments $x_{a,t-1}$. The high-ability and low-ability agents are illustrated by dotted and solid lines. For a low-ability agent a, if her endowment $x_{a,t-1} <$

$f_{1,1}$, she does not invest in education. Otherwise she invests. However, for agents with endowments between $f_{1,1}$ and $\overline{x}_{1,\mathrm{II}}$, they can borrow and invest in this period, but the bequests that they leave will be less than their endowments and may fall below the threshold $f_{1,1}$, because borrowing is costly. As a result, if their children are also low-ability, they cannot afford investing anymore. Therefore, in the long-run, the bequests of low-ability tend to converge to two level: the educated and rich group R_1 or the uneducated and poor group P. Similarly, the bequests of high-ability tend to converge to two level: the educated and rich group R_2 or the uneducated and poor group P.

From the above section, agents with the same ability are divided into three groups, according to their endowments. Therefore, taking heterogeneous ability into account, there exist six groups of agents. Formally, I define the steady state of the dynamic model as follows:

Definition 6. *The steady state is the static equilibrium as defined in Definition 1 which also satisfies the following condition: for each agent, the bequest is constant over time, i.e.* $x_{a,t} = x_a$.

In each period, by letting $x_t = x_{t-1}$ for each group of agents, we can solve for the following three possible fixed points for the three groups of agents with the same ability according to (18):

$$\overline{x}_{b,\mathrm{I}} = \frac{(1-\theta)w_u\,(1+r)}{1-(1-\theta)(1+r)} \tag{20}$$

$$\overline{x}_{b,\mathrm{II}} = \frac{(1-\theta)\left[\tau\,(1+i)-w_{b,e}\right]}{(1-\theta)(1+i)-1} \tag{21}$$

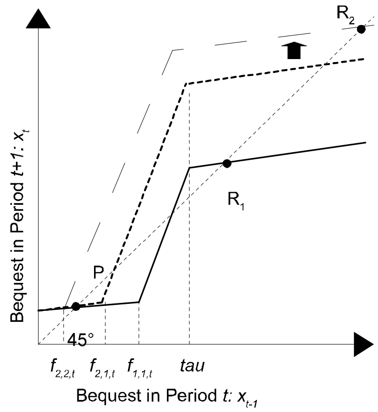

Figure 3: The Introducing of Diversified System

$$\overline{x}_{b,\mathrm{III}} = \frac{(1-\theta)\left[w_{b,e} - \tau\left(1+r\right)\right]}{1-(1-\theta)(1+r)} \tag{22}$$

In a steady state, the bequest of each agent must be equal to $\overline{x}_{b,\mathrm{I}}$, $\overline{x}_{b,\mathrm{II}}$ or $\overline{x}_{b,\mathrm{III}}$.[1] Otherwise, the bequest will still evolve, which violates the definition of the steady state. This logic allows to derive the main result on the dynamics in the following propositions, which do not depend on the initial distribution. The proofs are in the appendix.

Proposition 7. *(i) The dynamic model has two possible steady states: an egalitarian steady state where all the agents are educated or uneducated and the inequality is caused only by the heterogeneous ability; or a polarized steady state with where exits a rich and educated group and a poverty trap, which means that some poor agents are always uneducated, no matter they are high-ability or low ability.*

(ii) Transforming from a unified higher education system to a diversified one could not only increase the steady state wage of the educated, but also eliminate the poverty trap and lead to an egalitarian steady state where all agents are educated.

Proposition 7 (i) states that it is possible that there exists a poverty trap, which means that some agents and their children will be poor and uneducated forever, for example the poor group P illustrated in Figure 2. Because agents are constrained to their endowments, if the return to education is not high, those agents with low initial bequest, high-ability or low-ability, cannot afford investing in education and their children are going to stay being uneducated and poor. Moreover, if the return to education is very low, rich agents can invest in education. However, generation after generation, their bequests decrease and eventually, their later generations will not be able to invest, and all the agents stay poor and uneducated.

However, with the diversified higher education system introduced, as illustrated by the dashed , wage of educated high-ability agents becomes higher, which reduces $f_{2,2}$, the threshold for high-ability agents to invest. If the threshold is lower than $\overline{x}_{b,\mathrm{I}}$, the fixed point of the poor group, then poor agents always have a chance to get rid of the poverty trap and accumulate wealth by being high-ability and investing in education. The number of uneducated agents will be smaller and smaller until every agent is educated.

V. Conclusion

I analyze how socioeconomic composition affects the structure of the higher education system and the effects of the structure on the productivity and the inequality.

The analysis indicates that an economy with a larger population, a larger proportion of high-ability agents, higher return to high-tier education tends to have a diversified system. Compared with the unified system, the diversified system can add more productivity on high-ability agents and reduce the number of uneducated high-ability agents, which increases the aggregate productivity. The diversified system also increases the Gini coefficient but it is a Pareto improvement.

However, the analysis in this paper is carried out with the assumption of the uniform tuition fee, which is true in Germany and UK but not in the U.S. Also, higher education is privately funded in this paper. In the future work, I will compare privately funded higher education with publicly funded one.

VI. Appendix

A. *Proof of Proposition 2*

Profit for a university is:

$$y_u(1,1) = \tau \cdot N_u - \kappa_{e_u}$$

Thus, we have:

$$y(1,1) = \frac{1}{2}\tau \left\{\lambda L[1 - F(f_{1,1})] + (1-\lambda)L[1 - F(f_{2,1})]\right\} - \kappa_1$$

$$y(1,2) = \tau\lambda L[1 - F(f_{1,1})] - \kappa_1$$

$$y(2,1) = \tau(1-\lambda)L[1 - F(f_{2,2})] - \kappa_2$$

$$y(2,2) = \frac{1}{2}\tau(1-\lambda)L[1 - F(f_{2,2})] - \kappa_2$$

If $y(1,1) > y(2,1)$, it must be true that $y(1,2) > y(2,2)$. Therefore, there only exist three possible scenarios:

If $y(1,1) > y(2,1)$ and $y(1,2) > y(2,2)$, two universities operate in Tier 1: $\{e_w = 1, e_v = 1\}$. If $y(1,1) < y(2,1)$ and $y(1,2) > y(2,2)$, one university operates in each tier: $\{e_w = 1, e_v = 2\}$ or $\{e_w = 2, e_v = 1\}$. If $y(1,1) < y(2,1)$ and $y(1,2) < y(2,2)$, two universities operate in Tier 2: $\{e_w = 2, e_v = 2\}$.

B. *Proof of Proposition 3*

Let

$$G = y(1,1) - y(2,1)$$

Then we have

$$\frac{\partial G}{\partial \lambda} > 0$$

Also

$$\frac{\partial G}{\partial L} = \frac{1}{2}\tau \left\{\lambda[1-F(f_{1,1})] + (1-\lambda)[1-F(f_{2,1})] - 2(1-\lambda)[1-F(f_{2,2})]\right\} - (\kappa_2 - \kappa_1)$$

Therefore, $\frac{\partial G}{\partial L} < 0$ when

$$\lambda < \overset{\circ}{\lambda} = \frac{2\left(\kappa_2 - \kappa_1\right)/\tau - [1-F(f_{2,1})] + 2[1-F(f_{2,2})]}{[1-F(f_{1,1})] - [1-F(f_{2,1})] + 2[1-F(f_{2,2})]}$$

Also

$$\frac{\partial G}{\partial Z_{2,2}} < 0$$

C. Proof of Proposition 4

With a unified system, average human capital is

$$\overline{z}_t^u = (Z_{1,1}L_{1,1,t}^u + Z_{2,1}L_{2,1,t}^u + L_{u,t}^u)/L$$

With a unified system, average human capital is

$$\overline{z}_t^d = (Z_{1,1}L_{1,1,t}^d + Z_{2,2}L_{2,2,t}^d + L_{u,t}^d)/L$$

Then

$$\overline{z}_t^d - \overline{z}_t^u = \frac{Z_{2,2}L_{2,2,t}^d - Z_{2,1}L_{2,1,t}^u - (L_{u,t}^u - L_{u,t}^d)}{L}$$

Since $L_{u,t}^u - L_{u,t}^d = L_{2,2,t}^d - Z_{2,1}L_{2,1,t}^u$ $Z_{2,2} > Z_{2,1}$ and $L_{2,2,t}^d > L_{2,1,t}^u$, we have

$$\begin{aligned}\overline{z}_t^d - \overline{z}_t^u &= \frac{(Z_{2,2}-1)L_{2,2,t}^d - (Z_{2,1}-1)L_{2,1,t}^u}{L} \\ &> 0\end{aligned}$$

Therefore, the diversified system could bring higher average human capital.

D. Proof of Proposition 5

(in progress)

E. Proof of Proposition 7

Since $(1-\theta)(1+i) > 1$, there are only three possible fixed points: The uneducated fixed point, and fixed points for educated low-ability and high-ability agents.

(23)
$$\overline{x}_{\mathrm{I}} = \frac{(1-\theta)w_u(1+r)}{1-(1-\theta)(1+r)}$$

(24)
$$\overline{x}_{1,\mathrm{III}} = \frac{(1-\theta)\left[w_{1,1} - \tau(1+r)\right]}{1-(1-\theta)(1+r)}$$

(25)
$$\overline{x}_{2,\mathrm{III}} = \frac{(1-\theta)\left[w_{2,e} - \tau(1+r)\right]}{1-(1-\theta)(1+r)}$$

For all three points to exist at the same time, the following condition must hold: $\overline{x}_{\mathrm{I}} < f_{2,e} < f_{1,1} < \overline{x}_{1,\mathrm{III}} < \overline{x}_{2,\mathrm{III}}$, which is feasible. Specifically, the condition for $\overline{x}_{\mathrm{I}}$ existing is:

$$w_u < \frac{\theta + \theta r - r}{(i - \theta - \theta i)(1 + r)} \left[h(1 + i) - w_{2,e}\right]$$

If the higher education system is transformed from a unified one to a diversified one, $w_{2,e}$ is increased, which is possible to violated the condition above, which means that the uneducated fixed point stop existing.

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