# Inequality, Technological Change, and the Dynamics of the Skill Premium*

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Job Market Paper

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## Abstract

In this paper, I study different types of technological changes as explanations for the U-shape evolution of the skill premium observed in the U.S., throughout the 20th century. Technological change, skill-biased or unskill-biased, affects the skill premium directly by affecting the productivity and the demand for skilled and unskilled labour. Moreover, the distribution of income affects future cohorts' supply of skilled and unskilled labour. Therefore, technological change can also affect the skill premium and the inequality indirectly in the long-run, since it affects market wages and the transition of the distribution of income. The unskill-biased technological change at the beginning of the 20th century caused the decline in the skill premium in the first half of last century. The sustaining skill-biased technological change has continued to increase the skill premium since the midpoint of last century. However, this paper predicts that in the long-run, skill-biased technological change has an indirect dampening effect on the skill premium, which implies that skill-biased technological change could generate a Kuznets curve of the skill premium.



**Key Words:** skill premium, credit market imperfection, technological change, inequality, dynamic distribution of wealth

**JEL classification:** E24, I26, J24, O15, O33

# 1 Introduction

There has been renewed interest in the issue of the inequality of income and its dynamics (e.g. Piketty, 2014). Empirical evidence shows that wage inequality, especially the skill premium in the labour market is one of the main sources of income inequality.[1] Technological change is widely considered to govern the evolution of the skill premium, but in most of the existing literature, technological change only affects the skill premium directly by shifting the demand for skilled and unskilled workers. In this paper, by taking into consideration credit market imperfections, I allow for the possibility that technological change also affects the skill premium through the supply side of the labour market. With imperfect capital markets, the distribution of wealth affects investments in human capital and thus, the supply of different skills in the labour market. This in turn affects the skill premium and therefore, the distribution of wealth of the next generation. In an OLG model I analyze the long-run interaction between the skill premium and the distribution of wealth. Crucially, when technological change affects the skill premium directly, it also affects the transition of the distribution of wealth, thereby changing the supply of skills in future periods. Considering this subtle distributional effect yields richer dynamics of the skill premium. Moreover, this paper examines the effects of two types of technological changes on the skill premium: unskill-biased and skill-biased technological changes, and predicts that skill-biased technological change increases skill premium directly, but decreases it indirectly in the long-run.

Indeed, this model can provide some explanations for both cross-country and over-time patterns of the skill premium. As shown in Table 1, generally, the skill premium is larger in poorer countries. It is well known that this is easily explained by a static version of the model, which predicts that an economy with a higher cost of education, more severe credit market imperfection, and lower average income will have a larger skill premium. However, the skill premium does evolve over time. As in Table 2, stated by Goldin and Katz (2008), the skill premium in the U.S. experienced a non-monotonic evolution in the 20th century. The

[1] See, for example, Kijima (2006), and Lustig, Lopez-Calva and Ortiz-Juarez (2012).

skill premium declined in the first half of the century, and then it increased dramatically in the second half of the century, except the decline which occured in the 1970.[2] The dynamic version of the model provided in this paper can explain this evolution as a result of the unskill-biased technological change in the first half of the 20th century, the skill-biased technological change since the midpoint of the century, and the increased financial aid for college from the government in the 1970s.

Table 1. Mincerian Mean Rate of Return

| Country | Mincerian coefficient |
|---|---|
| Low income ($610 or less) | 11.2 |
| Lower middle income (to $2,449) | 11.7 |
| Upper middle income (to $7,619) | 7.8 |
| High income ($7,620 or more) | 6.6 |
| | |
| World | 10.1 |

Source: Psacharopoulos (1994)

Table 2 Changes in College/Non-College Log Relative Wages in the U.S.

| Years | 100 * Annual Log Changes |
|---|---|
| 1915-1940 | -0.56 |
| 1940-1950 | -1.68 |
| 1950-1960 | 0.83 |
| 1960-1970 | 0.69 |
| 1970-1980 | -0.74 |
| 1980-1990 | 1.51 |
| 1990-2000 | 0.58 |
| 2000-2005 | 0.50 |

Source: Goldin and Katz (2008)

[2] A Similar U-shaped pattern in the same period can be found in the data of wage of craftsmen relative to that of labours in England (Clark (2005)).

This paper analyzes a labour market, which has endogenous demand and endogenous supply of two types of workers: skilled and unskilled. Because of credit market imperfection, individuals can only borrow a limited amount in order to finance their investment in education, as in other inequality and growth models (Galor and Zeira (1993); Banerjee and Newman (1993)). Education investment thus depends on the distribution of endowments, which in turn, depends on past wages. Moreover, firms' choice of production technology is endogenous as well, and depends not only on the access to a technology, but also on other factor endowments of the economy. For example, given access to the same technology, firms from a skilled-labour abundant country and firms from an unskilled-labour abundant country will choose different technologies. The choice of technology is modeled using the idea of a technology frontier proposed by Caselli and Coleman (2006). Caselli and Coleman define the choice of technology as the choice of productivities of different types of workers. In this model, firms hire skilled and unskilled workers and choose productivities for them simultaneously. The technology frontier is the set of all non-dominated feasible technology choices, from which a firm in a certain country can choose. Technological change can be viewed as the shifting of the technology frontier. Skill-biased technological change, for example the invention of the computer, makes it cheaper for firms to increase productivity for skilled workers. Unskill-biased technological change, for example the invention of the assembly line, makes it cheaper for firms to increase productivity for unskilled workers. They can be considered as the expansion of the technology frontier in different dimensions.

In each period, individuals decide on their education and become skilled or unskilled workers. Firms individually choose their production technology from a set of all feasible technologies and hire factors given their rental rates. Given the distribution of wealth, the static equilibrium is an allocation of workers, their education choices, and firms' choice of production technology that clears the labour market. The analysis of the static equilibrium generates a first result that an economy tends to have a larger skill premium if the cost of education is higher, the credit market imperfection is more severe, the average wealth is higher, and it is cheaper to increase productivity for skilled workers.

In the long-run, wages affect the wealth and bequests of individuals, and the skill premium interacts with the distribution of wealth. To determine the long-run dynamics of the model, I solve for a steady state which can only be one of two different cases. In the first type of steady state, the egalitarian steady state, the wealth of every individual converges to the same level, the skill premium is small, credit market imperfections play no role, and individuals are indifferent in regards to becoming skilled or not. In the other type of steady state, the polarized steady state, there are two different long-run wealth levels, the skill premium is large and credit market imperfections affect the supply of skills. Which steady state is attained is determined by the initial status of the economy. Generally, an economy with less severe credit market imperfections and cheaper productivity of unskilled workers has an egalitarian steady state.

The exogenous technological change affects the transition to the steady state. When technological change affects the skill premium directly in the current period, it also affects the transition of the distribution of wealth and therefore, the skill premium in the long-run. Unskill-biased technological change decreases the skill premium directly, and if the change is large enough, it also causes the distribution of wealth to transit to an egalitarian steady state. The reason is that unskill-biased technological change makes it cheaper to increase productivity for unskilled workers and expands the technology frontier. As a result, the real wages of both skilled and unskilled workers are increased. Therefore, unskilled workers cannot afford education but they accumulate wealth. Some of their children will gain an education and become skilled. An increase in the supply of skilled workers decreases the skill premium further and the distribution of wealth transit to an egalitarian steady state where every agent has the same wealth. However, the direct and indirect effects of skill-biased technological change on the skill premium are different. Skill-biased technological change increases the skill premium directly because it encourages firms to increase productivity for skilled workers. However, this change also expands the technology frontier and increases the real wages of skilled and unskilled workers. If the skill-biased technological change is large enough, the incomes of unskilled workers converge to a higher level in the next period, which increases the supply of skilled workers as fewer individuals are credit constrained. The distribution of wealth transits to an egalitarian steady state and has a dampening effect on the skill premium. Therefore, a large enough skill-biased technological change generates a Kuznets curve of the skill premium: it increases the skill premium when it happens, but also causes the distribution of wealth to follow an egalitarian transition; when it stops, the egalitarian transition decreases the skill premium.

The previous analysis provides some explanations for the non-monotonic evolution throughout the 20th century. The technological change at the beginning of the 20th century was unskill-biased. Mass production and assembly lines replaced skilled workers and broke down the production process into a series of elementary tasks that could be performed by unskilled workers. This change encouraged firms to increase productivity for unskilled workers, which increased the unskilled wage and decreased the skill premium. According to the previous analysis, both the direct and the indirect effects of unskill-biased technological change decrease the skill premium. This explains the decline in the skill premium in the first half of the 20th century. However, technological change has been skill-biased since the midpoint of last century. The new technology, computers and automatons for example, replaced unskilled workers and encouraged firms to increase productivity for skilled workers. Autor, Katz and Krueger (1998) suggest that the growth of relative demand for skilled workers, which could be largely explained by the spread of computer technology, was still rapid in 1995, which implies that the skill-biased technological change was still in progress at the end of last century. The increase in the skill premium in the second half of the 20th century was a result of the direct effect of the sustaining skill-biased technological change. Moreover, we can predict that if the current skill-biased technological change is large enough, when it stops, the skill premium will decrease since the distribution of wealth will follow an egalitarian transition.

This paper differs from previous literature in the following respects. By introducing the endogenous supply of workers and credit market imperfection into Caselli and Coleman's (2006) model, it analyzes the dynamic distribution of wealth as a new channel, through which technological change affects the skill premium. The dynamic model allows the skill premium and the distribution of wealth to interact with each other. Firstly, technological change can not only affect the skill premium through affecting the demand side of the labour market, i.e. affecting the hiring choice of producers, but also affect the supply side of the labour market by affecting the distribution of wealth. Secondly, the model can analyze both the short-run and the long-run effects of technological change on the skill premium. Moreover, compared with Galor and Zeira (1993), the endogenous technology allows us to analyze how the choice of technology in an economy is affected by the distribution of wealth, credit market imperfection and other factors, besides the access to technology.

The existing literature offers several explanations for the difference of the skill premium across countries. Perhaps most significantly, Acemoglu (1999) states that technological change could increase wage inequality directly and could also increase it indirectly by changing the structure of the labour market. Blau and Kahn (1996) and Acemoglu (2003) use institutional differences to explain the differences of the skill premium. Specifically, Acemoglu (2003) finds that European labour market institutions compress wage inequality and also encourage the upgrading of unskill-biased technology, which reduces the wage inequality further. Moreover, since income inequality can be largely explained by the skill premium, this paper is also related to the literature on financial market imperfections, inequality and growth, emphasizing the role of wealth distribution in determining the supply of skilled labour. For example, Beck, Demirguc-Kunt and Levine (2005), find that financial development reduces income inequality. Gall, Schiffbauer, and Kubny (2014) also discuss the effect of credit market imperfection on FDI and inequality. Gregorio and Lee (2002) indicate that higher educational attainment and more equal distribution of education reduce the income inequality. Furthermore, Guvenen, Kuruscu and Ozkan (2013) state that labour income taxation affects wage inequality. This literature tends to abstract from technological change.

Skill-biased technological change is widely used to explain the evolution of the labour market of the U.S. for example, as in Autor and Dorn (2013). Particularly, Acemoglu (1998) and Galor and Moav (2000) analyze the skill premium with endogenous technological change. However, unlike this paper, most papers focus on the effect of technological change on the demand side of the labour market. This paper shares a similar argument with Galor and Moav (2000) specifically that the expansion of financial aid for college can explain the fall of the skill premium in the 1970s. There are, of course, other factors that may affect wage premium and inequality: e.g. international trade, as discussed in Wood (1995) and Acemoglu (2003). Other studies, for example, Card and Shleifer (2009) examine the effects of immigration on wage inequality in the U.S.

The remainder of this paper is organized as follows: in Section 2, I set up the basic model. Section 3 examines the static equilibrium. Section 4 discusses the dynamic version of the model and its steady states. Section 5 offers some concluding remarks.

# 2 The Model

We consider a small open economy. The economy is populated by a continuum of agents with a mass of one, which is constant over time.

There is a continuum of competitive firms, which hire three factors to produce: capital, skilled workers and unskilled workers. Skilled and unskilled workers are substitutes to each other. Firms can also choose productivities for both types of workers simultaneously, from the set of all feasible technology choices.

A single good is produced by firms and consumed by agents. In this small open economy, the price of the good is equal to the world market price, which is exogenous and normalized to 1. Agents can save and borrow in order to finance their education, and firms can borrow to finance production in an international credit market. Both the saving and borrowing world market interest rates are also exogenous and constant over time.

## *2.1 Imperfect Credit Market*

The credit market is imperfect, in that there is a spread between the risk free saving and borrowing interest rates denoted by $r$ and $i$, respectively. Set the spread be denoted by $\beta \geq 0$, so that $1 + i = \beta(1 + r)$. This assumption is borrowed from that of Galor and Zeira (1993), which is a tried and tested way in incorporating borrowing constraints. Borrowing constraints are more severe for households than for firms. For simplicity, I assume that firms can borrow at rate $r$.

## *2.2 Households*

Each agent lives for two periods in overlapping generations: young and old. In period $t$, a young agent receives bequest $x_{a,t}$ from her parent and decides on her education: she has the choice either to invest in education or not. The cost of investing in human capital is $h$. When old, agents work as skilled or unskilled workers, depending on their education level, and earn skilled or unskilled wages $w_{u,t}$ or $w_{s,t}$. Agents only consume and leave bequests to children in the second period of their life. This framework closely follows Galor and Zeira (1993). Agent $a$ receives lifetime utility $u_{a,t}$ from both consumption $c_{a,t}$ and the bequest $x_{a,t}$:

$$u_{a,t} = \theta \ln c_{a,t} + (1 - \theta) \ln x_{a,t} \tag{1}$$

The parameter $\theta \in (0, 1)$ determines the saving rate. The optimal choice of $c_{a,t}$ and $x_{a,t}$ for an individual $a$ in the second period of her life maximizes $u_{a,t}(c_{a,t}, x_{a,t})$ subject to the budget constraint:

$$c_{a,t} + x_{a,t} = \pi_{a,t} \tag{2}$$

where $\pi_{a,t}$ is lifetime income. Therefore the individual utility only depends on lifetime income $\pi_{a,t}$ and is strictly increasing in the income. As a result, the agent's problem is to maximize lifetime income by deciding on her education.

An agent working as an unskilled worker without investing has income

$$\pi_{a,t} = (w_{u,t} + x_{a,t-1})(1 + r) \tag{3}$$

An agent with bequest $x_{i,t-1} \geq h$, who invests in human capital, obtains:

$$\pi_{a,t} = w_{s,t} + (x_{a,t-1} - h)(1 + r) \tag{4}$$

An agent, who receives bequest $x_{i,t-1} < h$ and invests, needs to borrow and has income:

$$\pi_{a,t} = w_{s,t} - (h - x_{a,t-1})(1 + i) \tag{5}$$

All the agents with bequest $x_{i,t-1} \geq h$ prefer to invest and work as skilled workers if the following assumption holds:

$$w_{s,t} - h(1 + r) \geq w_{u,t}(1 + r) \tag{6}$$

If this assumption is violated, all individuals work as unskilled and there is an excess supply of unskilled workers. This drives the unskilled wage down and the skilled wage up until (6) is satisfied.

Hence an agent is indifferent between investing and not investing if $\pi_{a,t}(invest) = \pi_{a,t}(not\ invest)$, which pins down an endowment $f_t$ :

$$f_t = \frac{1}{i - r}[w_{u,t}(1 + r) + h(1 + i) - w_{s,t}] \tag{7}$$

Therefore, given wages, $w_{s,t}$ and $w_{u,t}$, all agents with endowment greater than $f_t$ will invest in human capital and agents with endowment smaller than $f_t$ will not invest. If the distribution of the bequest in period $t$ is $D_t(x_a)$, then the supply of different workers is:

$$L^S_{u,t}(w_{s,t}, w_{u,t}) = \int_0^{f_t} dD_t(x_a) \tag{8}$$

$$L^S_{s,t}(w_{s,t}, w_{u,t}) = \int_{f_t}^{\infty} dD_t(x_a) = 1 - L^S_{u,t}(w_{s,t}, w_{u,t}) \tag{9}$$

## *2.3 Firms*

In period $t$, a representative firm generates output using the production function proposed by Caselli and Coleman (2006):

$$y_t = k_t^{\alpha} \left[ (A_{u,t} L_{u,t})^{\sigma} + (A_{s,t} L_{s,t})^{\sigma} \right]^{(1-\alpha)/\sigma} \tag{10}$$

Three factors are used to produce: capital $k_t$, the ratio of unskilled workers $L_{u,t}$, and the ratio of skilled workers $L_{s,t}$, with $L_{u,t}+ L_{s,t} = 1$. $A_{u,t}$ and $A_{s,t}$ are the productivities of two types of workers $\alpha \in (0, 1)$. $1/(1 - \sigma)$ is the elasticity of substitution between skilled and unskilled workers and $\sigma \in (0, 1)$.

To maximize its profit, taking wages $w_{s,t}$, $w_{u,t}$ and borrowing interest rate $r$ as given, a representative firm optimally chooses factor inputs $k_t$, $L_{u,t}$ and $L_{s,t}$. A firm also chooses the production technology ($A_{u,t}$, $A_{s,t}$) from a set of feasible technology choices in that period. This set is given by:

$$\delta (A_{s,t})^{\omega} + \gamma (A_{u,t})^{\omega} \leq B \tag{11}$$

In Figure 1, $\Phi_1$, $\Phi_2$ and $\Phi_3$ illustrate three different technology frontiers, i.e. the sets of all non-dominated ($A_{u,t}$, $A_{s,t}$) pairs. Parameters $\omega$, $\delta$, $\lambda$ and $B$ are exogenous and strictly positive. Parameters $\omega$ and $\gamma$ measure the trade-off between the productivities of skilled and unskilled workers. Both $\mathcal{B}$, $\delta$ and $\gamma$ differ across economies while $\omega$ is identical for all the economies.[3]

[3]Caselli and Coleman (2006) prove that the assumption $\omega > \sigma/(1 - \sigma)$ needs to hold to rule out the

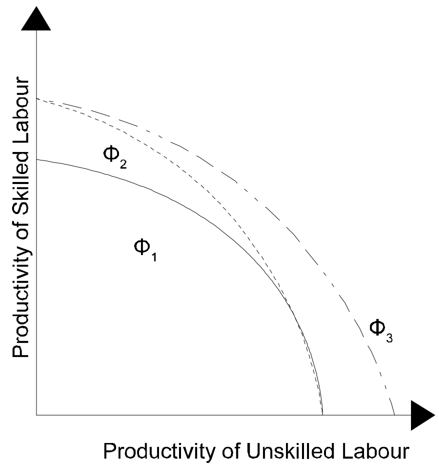


Figure 1: Technology Frontier

The parameter $B$ denotes the level of the technology frontier of a country, which represents total factor productivity. Therefore, an increase in $B$ represents an unbiased (balanced) technological change, shown as a shift of the frontier from $\Phi_1$ to $\Phi_3$ in Figure 1.

The parameters $\delta$ and $\gamma$ denote the relative prices of productivity of unskilled workers and productivity of skilled workers. A decrease in $\delta$ represents skill-biased technological change. For example, one can argue that the invention of the computer made it less costly to increase the productivity of skilled workers. This can be represented by a decreased $\delta$ in the frontier, and a shift from $\Phi_1$ to $\Phi_2$ in Figure 1. Then, given this change, firms applied computers in production, which can be shown as firms adjusting their choices of $(A_{u,t}, A_{s,t})$. Similarly, a decrease in $\gamma$ represents unskill-biased technological change. An example of this type of technological change is the invention of the assembly line, which made it less costly to increase the productivity of skilled workers. An unskill-biased technological change can be shown as a shift from $\Phi_2$ to $\Phi_3$ in Figure 1.

A firm chooses technology and factor input to solve:

situation that the supply of labour is mixed but some firms always choose to set $A_{u,t} = 0$ and only hire skilled workers and other firms do the opposite.

$$\underset{k_t, L_{u,t}, L_{s,t}}{Max} \quad k_t^{\alpha} \left[ \left( A_{u,t}^* L_{u,t} \right)^{\sigma} + \left( A_{s,t}^* L_{s,t} \right)^{\sigma} \right]^{(1-\alpha)/\sigma} - rk_t - w_{s,t} L_{s,t} - w_{u,t} L_{u,t} \tag{12}$$

subject to:

$$\delta \left( A_{s,t} \right)^{\omega} + \gamma \left( A_{u,t} \right)^{\omega} \leq B$$

The following can be derived from the first order conditions for problem (12):

$$\frac{L_{s,t}^D}{L_{u,t}^D} = \left( \frac{\gamma}{\delta} \right)^{\frac{\sigma}{\omega - \sigma - \omega\sigma}} \cdot \left( \frac{w_{s,t}}{w_{u,t}} \right)^{\frac{\omega - \sigma}{\omega\sigma - (\omega - \sigma)}} \tag{13}$$

Equation (13) shows firms' relative demand for different workers according to the relative wage, when it can adjust technology along the technology frontier.

# 3 Static Equilibrium

In each period, both the price for the good and the interest rates are exogenous, and the static equilibrium is an allocation of workers and investment choices that clears the labour market. Formally,

Definition 1. *A static equilibrium is an allocation of factors ($L_{u,t}, L_{s,t}, k_t$), a technology ($A_{u,t}, A_{s,t}$), and prices ($w_{u,t}, w_{s,t}$), such that in period $t$, for given distribution of endowments $D_t(x_a)$ and other parameters ($\alpha, \sigma, h, i, r, \theta, \mathcal{B}, \delta, \gamma, \omega$):*

*1. ($A_{u,t}, A_{s,t}$) satisfies feasibility: the technology frontier (11) holds;*

*2. (8) and (9), yield labour supply $L_{u,t}^S$ and $L_{s,t}^S$, so that the utility of each agent is maximized;*

*3. ($L_{u,t}^D, L_{s,t}^D, A_{u,t}, A_{s,t}, k_t$) solve the problem of the representative firm (12);*

*4. the labour market clears:*

$$L_{u,t}^S(w_{u,t}, w_{s,t}) = L_{u,t}^D(w_{u,t}) \tag{14}$$

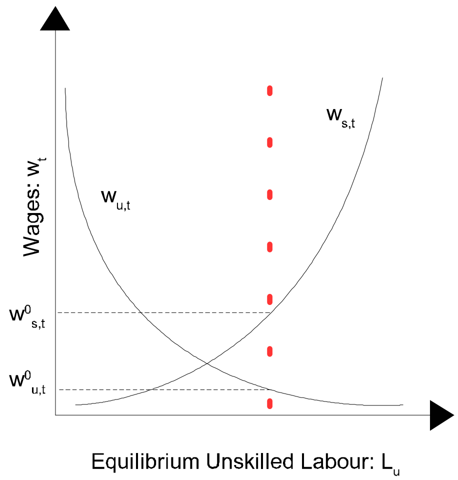


Figure 2: Equilibrium Labour Ratio and Wages

$$L^S_{s,t}(w_{u,t}, w_{s,t}) = L^D_{s,t}(w_{s,t}) \tag{15}$$

To examine the static equilibrium, firstly let's imagine that the supply of skills is exogenous. Then, firms maximize their profit by deciding on technology, given the supply of each type of worker. The wage for each type of worker will be equal to the marginal productivity of that type. Hence as illustrated in Figure 2, given any possible unskilled labour ratio $L_{u,t}$, there is a corresponding pair of wages.

Lemma 2. *As the supply of unskilled labour $L_{u,t}$ increases from 0 to 1, unskilled wage $w_{u,t}$ decreases from infinity, and skilled wage $w_{s,t}$ increases to infinity.*

The proof of the lemma is in the appendix. Then I take the endogenous supply of skills into account and we can find that not all the values of $L_{u,t}$ are feasible in the static equilibrium. The reason is that, as illustrated in Figure 2, on the left-hand-side of the dotted line, compared with $w_{u,t}$, $w_{s,t}$ is not large enough to ensure that the condition (6) holds, which means in this situation the return to the education is very low so that even the agents who do not need to borrow will not invest in education and the supply of skilled workers is equal to zero. As a result, the equilibrium always happens on the right-hand-side of the dotted line. Therefore, we have the following lemma and the proof is in the appendix.

LEMMA 3. *Only one unique pair* $(w^0_{u,t}, w^0_{s,t})$ *make agents who do not need to borrow indifferent between investing or not, i.e. make*

$$w^0_{s,t} - h(1+r) = w^0_{u,t}(1+r) \tag{16}$$

*hold, and the static equilibrium skill premium* $\frac{w^*_{s,t}}{w^*_{u,t}} \geq \frac{w^0_{s,t}}{w^0_{u,t}}$.

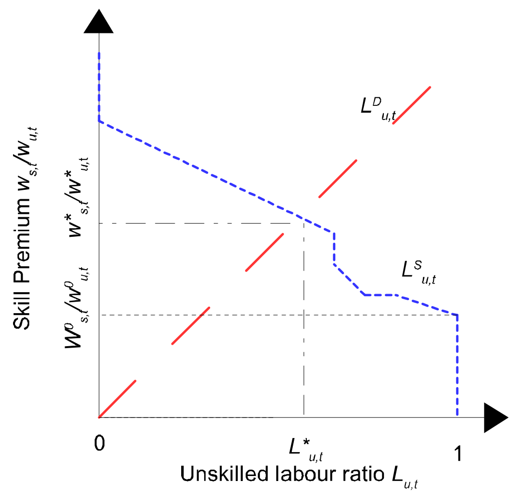


Figure 3: Static Equilibrium

Figure 3 illustrates the static equilibrium. In Figure 3, as illustrated by the dashed line, the demand for unskilled workers, $L^D_{u,t}$, is strictly monotonically increasing as the skill premium increases. The supply curve of unskilled workers, illustrated by the dotted line, consists of three parts. The first part corresponds to $\frac{w_{s,t}}{w_{u,t}} < \frac{w^0_{s,t}}{w^0_{u,t}}$, i.e. the skill premium is so low that it is not profitable for anyone to invest, and the supply of unskilled workers is $L^S_{u,t} = 1$. The third part corresponds to a very high skill premium, so that $f_t$ is smaller than the smallest bequest, and every agent is willing to invest, which means that the supply of skilled workers is $L^S_{u,t} = 0$. Between these two parts, $L^S_{u,t}$ decreases in the skill premium. The intersection of $L^D_{u,t}$ and $L^S_{u,t}$ defines the unique static equilibrium labour proportions $(L^*_{u,t}, L^*_{s,t})$ and wages $(w^*_{u,t}, w^*_{s,t})$. The second part of the supply curve is downward sloping but also contains horizontal and vertical segments. This is because if there is no agent with endowment between two levels of the threshold, the supply curve is vertical. Conversely, if a positive mass of agents have the same amount of endowment, the supply curve is horizontal.

There is a special case of the static equilibrium: every agent has the same bequest: $x_{a,t} = x_t, \forall a$. All individuals in this situation are indifferent between being skilled and unskilled. That is, the equilibrium wages $w^*_{u,t}$ and $w^*_{s,t}$ solve $x_t = f_t$, i.e. the bequest just equals the threshold value that is defined in Equation (7).

To derive a closed form solution, let us consider the following assumption:

Assumption 4. *At the beginning of a period $t$, the endowments follow a uniform distribution on $[M - \varepsilon, M + \varepsilon]$, with $M \in (0, \infty)$ and $\varepsilon \in (0, M)$.*

The parameter $M$ represents the average level of bequests and $\varepsilon$ represents endowment inequality. If $M - \varepsilon \leq f_t \leq M + \varepsilon$, then $L^S_{u,t} = (f_t - M + \varepsilon)/2\varepsilon$ and $L^S_{s,t} = (M + \varepsilon - f_t)/2\varepsilon$.

The following proposition states the comparative statics of the static equilibrium. This proposition can offer some explanations for the differences of the skill premium across countries. The proof is in the appendix.

Proposition 5. *(Comparative Statics)*

*(i) Suppose $\widehat{h} > h$, then the equilibrium labour ratio $\widehat{\frac{L_{s,t}}{L_{u,t}}} \leq \frac{L_{s,t}}{L_{u,t}}$, the equilibrium relative productivity $\widehat{\frac{A_{s,t}}{A_{u,t}}} \leq \frac{A_{s,t}}{A_{u,t}}$, and the equilibrium skill premium $\frac{\widehat{w_{s,t}}}{\widehat{w_{u,t}}} \geq \frac{w_{s,t}}{w_{u,t}}$, for all $h \in (0, \infty)$.*

*(ii) Suppose $\widehat{\beta} > \beta$, then the equilibrium labour ratio $\widehat{\frac{L_{s,t}}{L_{u,t}}} \leq \frac{L_{s,t}}{L_{u,t}}$, and the equilibrium relative productivity $\widehat{\frac{A_{s,t}}{A_{u,t}}} \leq \frac{A_{s,t}}{A_{u,t}}$, and the equilibrium skill premium $\frac{\widehat{w_{s,t}}}{\widehat{w_{u,t}}} \geq \frac{w_{s,t}}{w_{u,t}}$, for all $i \in (r, \infty)$.*

*(iii) Suppose $\widehat{M} > M$, then the equilibrium labour ratio $\widehat{\frac{L_{s,t}}{L_{u,t}}} \geq \frac{L_{s,t}}{L_{u,t}}$, the equilibrium relative productivity $\widehat{\frac{A_{s,t}}{A_{u,t}}} \geq \frac{A_{s,t}}{A_{u,t}}$, and the equilibrium skill premium $\frac{\widehat{w_{s,t}}}{\widehat{w_{u,t}}} \leq \frac{w_{s,t}}{w_{u,t}}$, for all $M \in$*

$(0, \infty)$.

*(iv) Suppose* $\widehat{\gamma} \geq \gamma$ or $\widehat{\delta} \leq \delta$, *then the equilibrium labour ratio* $\widehat{\frac{L_{s,t}}{L_{u,t}}} \geq \frac{L_{s,t}}{L_{u,t}}$, *the equilibrium relative productivity* $\widehat{\frac{A_{s,t}}{A_{u,t}}} \geq \frac{A_{s,t}}{A_{u,t}}$, *and the equilibrium skill premium* $\frac{\widehat{w_{s,t}}}{\widehat{w_{u,t}}} \geq \frac{w_{s,t}}{w_{u,t}}$.

Proposition 5(i) is straightforward. A higher cost of investing makes education available to fewer agents, which reduces the supply of skilled workers and leads to a larger skill premium.

Proposition 5(ii) implies that if a country has more severe credit market imperfections, it is likely to have a larger skill premium. If the credit market imperfection is severe, fewer agents can invest in education by borrowing. This leads to a decrease in the supply of skilled workers. A similar argument is discussed by Banerjee and Newman (1993), and Galor and Moav (2000). This argument is supported empirically by Li, Squire and Zou (1998), who find that there is a positive relation between income inequality and the imperfections of the credit market.

Proposition 5(iii) is best interpreted as that in wealthier countries the proportion and productivity of skilled workers are higher, but the skill premium is lower, which means the income is more equal. This is because with more wealth, more agents can afford to invest in education, which leads to an increase in the supply of skilled workers. This result offers an explanation for the positive relation between the skill premium and GDP per capita in most countries, just as shown in Table 1. Caselli and Coleman (2006) also offer estimation results, showing that higher-income countries are skilled labour abundant and use skilled labour more efficiently than lower-income countries. Considering the skill premium as an important reason for income inequality, similar evidence can also be found in the work of Lindert and Williamson (1985), who find that the right portion of the Kuznets Curve is more robust, which means that the inequality falls as the per capita income increases at higher levels of development.

Proposition 5(iv) states that the skill premium tends to be larger in countries where the skill-biased technology is relatively cheaper. Firms are therefore more willing to increase the relative productivity of skilled workers and hire more of them. The increased relative productivity also leads to a higher skill premium. This argument is similar to Acemoglu's (1999): skilled-biased technology increases the wage inequality, independently of whether it leads to a change in the structure of the labour market or not. Autor, Katz and Krueger (1998) also find that demand for college graduates grew more rapidly on average from 1970 to 1995, which can be explained largely by the spread of computer technology. This proposition also indicates the direct effect of technological change: unskill-biased one decreases the skill premium and skill-biased one increases the skill premium.

# 4 The Dynamic Model

In this section, I develop the model to a dynamic version, by taking into account the transition of the distribution of wealth. Wages determine the income of each agent and then the bequest she gives to her child. As a result, in the long-run, the distribution of wealth becomes endogenous as well. To examine the long-run evolution of the economy and its skill premium, I firstly characterize its steady state, and then analyze how the exogenous shocks affect the skill premium in the long-run, by affecting the transition.

## *4.1 Steady State Cases*

From the above section, we know that all the agents can be divided into three groups according to their investment decisions: agents from Group I will not invest because their bequest is lower than the threshold; agents from Group II will invest and borrow because their endowments are higher than the threshold but lower than the cost of education; agents from Group III will invest in education without borrowing, because they receive bequests higher than the cost of education. Bequests of agents evolve as follows:

$$x_{a,t} = \begin{cases} (1-\theta)[(w_{u,t} + x_{a,t-1})(1+r)], if\, x_{a,t-1} < f_t & (Group\, I) \\ (1-\theta)[w_{s,t} - (h - x_{a,t-1})(1+i)], if\, f_t \leqslant x_{a,t-1} < h & (Group\, II) \\ (1-\theta)[w_{s,t} + (x_{a,t-1} - h)(1+r)], if\, x_{a,t-1} \geqslant h & (Group\, III) \end{cases} \tag{17}$$

I suppose that $(1-\theta)(1+r) < 1$ to focus on interesting dynamics, following Galor and Zeira (1993). This assumption rules out the possibility that the incomes of agents in Group I converge to zero or the incomes of agents in Group III diverge. Formally, I define the steady state of the dynamic model as follows:

Definition 6. *The steady state is the static equilibrium as defined in Definition 1 which also satisfies the following conditions: in each period, wages* $(w_{u,t}, w_{s,t})$ *are equal to a constant pair* $(w_u, w_s)$*; for each agent, the bequest is constant over time, i.e.* $x_{a,t} = x_a$.

In each period, by letting $x_t = x_{t-1}$ for each group of agents, we can solve for the following three possible fixed points for the three groups according to (17):

$$\overline{x}_{\mathrm{I,t}} = \frac{(1-\theta)w_{u,t}\,(1+r)}{1-(1-\theta)(1+r)} \tag{18}$$

$$\overline{x}_{\mathrm{II,t}} = \frac{(1-\theta)\,[h\,(1+i) - w_{s,t}]}{(1-\theta)(1+i) - 1} \tag{19}$$

$$\overline{x}_{\mathrm{III,t}} = \frac{(1-\theta)\,[w_{s,t} - h\,(1+r)]}{1-(1-\theta)(1+r)} \tag{20}$$

These three possible fixed points determine the dynamics of the distribution of wealth. For example, Figure 4 illustrates the dynamics of distribution, i.e. given endowments, how much bequest agents from three group are going to leave to children, when $(1-\theta)(1+i) < 1$. The intercept of the segment representing Group I is governed by the wage of unskilled workers. The intercept of the segment representing Group III is governed by the wage of

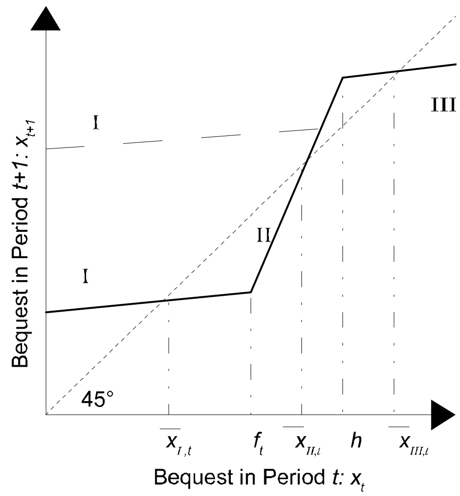


Figure 4: Transition of the Distribution of Wealth

skilled workers. Thus, the distance between two segments indicates the skill premium. In the dynamic illustrated by the solid line, agents with endowments $x_{t-1} < f_t$ will be unskilled and their bequests $x_t$ converge to $\overline{x}_{I,t}$. Agents with endowments $x_{t-1} \geq \overline{x}_{II}$ will be skilled and their bequests $x_t$ converge to $\overline{x}_{III,t}$. Agents with endowments $f_t \leq x_{t-1} < \overline{x}_{II}$ will be skilled. But because borrowing is costly, their bequests $x_t$ also converge to the low level $\overline{x}_{I,t}$. In the next period, the skill premium will be larger since fewer agents can afford education. However, in the dynamic with Group I illustrated by the dash line, the wage of unskilled workers is not very low. Thus, unskilled workers accumulate wealth and the bequests of all the agents converge to $\overline{x}_{I,t}$. In the next period, the skill premium will be smaller since more agents can afford education.

In a steady state, the bequest of each agent must be equal to $\overline{x}_I$, $\overline{x}_{II}$ or $\overline{x}_{III}$, which are the fixed points with the steady state wages.[4] Otherwise, the bequest will still evolve, which violates the definition of the steady state. This logic allows to derive the main result on the dynamics in the following propositions, which do not depend on the initial distribution. The proofs are in the appendix.

[4]It is not necessary that all these three points exist in the steady state.

Proposition 7. *In the steady state,*

*if* $(1-\theta)(1+i) < 1$, *the model has an egalitarian steady state where all the agents have the same wealth;*

*if* $(1-\theta)(1+i) > 1$, *the model has two possible cases of steady state: an egalitarian steady state where all the agents have the same wealth and the skill premium is small, or a polarized steady state, where there are two unequal levels of wealth, and the skill premium is large.*

Proposition 8. *An economy with less severe credit market imperfection tends to reach an egalitarian steady state; an economy with more severe credit market imperfection tends to reach a polarized steady state.*

Proposition 7 states that there are only two possible cases of steady state. One possible case is that each agent has the same wealth and it is indifferent to being skilled or unskilled worker. The other case is that there exist two unequal levels of wealth. The agents in the rich group are skilled while the agents in the poor group are unskilled.

Proposition 8 is implied by Proposition 7. It states that the credit market imperfection not only affects the static equilibrium skill premium, but also affects the skill premium and the inequality of the economy in the long-run. The economy with a less severe credit market will have a smaller skill premium and an equal distribution of wealth, while the economy with a more severe credit market will have a larger skill premium and an unequal distribution of wealth.

## *4.2 Technological Changes and the Skill Premium in the the 20th Century*

In this subsection, I offer an example to show how technological changes affect the skill premium directly and indirectly in the long-run by changing the dynamic distribution of wealth. Furthermore, I argue that the effects of technological changes are possible explanations for the non-monotonic behaviour of the skill premium of the U.S. in the 20th century.

PROPOSITION 9. *When the distribution of endowment is continuous and* $(1-\theta)(1+i) > 1$,

*there exists a* $\overset{\circ}{\gamma}$, *if* $\gamma < \overset{\circ}{\gamma}$, *the model has an egalitarian steady state*;

*there exists a* $\overset{\circ}{\delta}$, *if* $\delta < \overset{\circ}{\delta}$, *the model has an egalitarian steady state.*

The proof of this proposition is in the appendix. This proposition states how technological change affects the evolution of the skill premium indirectly by affecting the transition of the distribution of wealth. In the long-run, both unskill-biased and skill-biased technological change could cause the distribution of wealth to transit to an egalitarian steady state and decrease the skill premium indirectly, no matter what type of transition the distribution follows before the technological change.[5]

Proposition 5 and Proposition 9 together state the direct and indirect effects of technological change on the skill premium. Unskill-biased technological change, represented by a decrease in $\gamma$, decreases the skill premium immediately, because it encourages firms to increase productivity for unskilled workers. Moreover, if the unskill-biased technological change is large enough, it could also make the distribution of wealth follow an egalitarian transition. In Figure 5, this change is indicated by the black arrows. Because the technology frontier is expanded, the real wages of both skilled and unskilled workers are increased. This leads the converging point of the unskilled group $\overline{x}_{\mathrm{I,t}}$ to be higher than the threshold of education $f_t$, which implies that the agents of the unskilled group will become richer and some of their children will gain an education and become skilled. An increase in the supply of skilled workers decreases the skill premium further and the distribution of wealth transits to an egalitarian steady state where every agent has the same wealth. This indirect effect is indicated by the white arrows in Figure 5.

However, the direct and indirect effects of skill-biased technological change on the skill premium are different. Skill-biased technological change, represented by a decrease in $\delta$,

[5] However, a large $\gamma$ or $\delta$ is not a sufficient condition for a polarized steady state. For example, if the initial endowment of every agent is higher than the cost of education, an egalitarian steady state will be obtained.

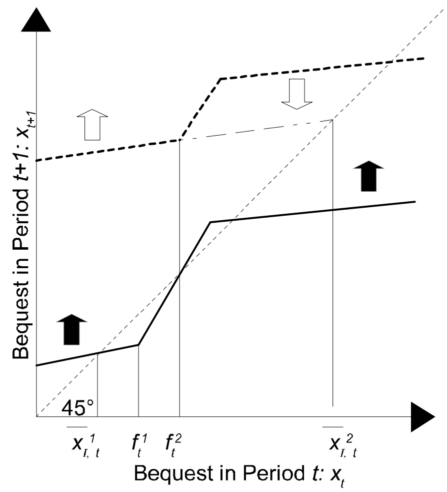


Figure 5: Unskill-biased Technological Change

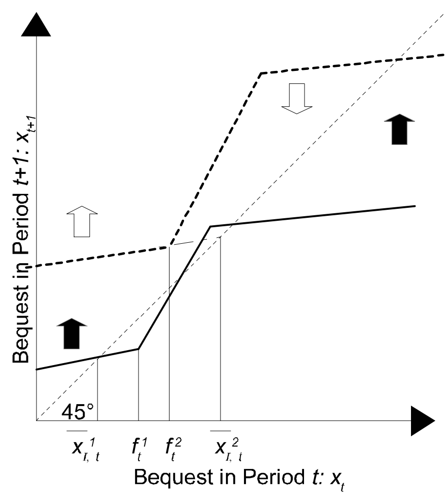


Figure 6: Skill-biased Technological Change

increases the skill premium immediately, because it encourages firms to increase productivity for skilled workers. But skill-biased technological change also expands the technology frontier and increases the real wages of skilled and unskilled workers. If the skill-biased technological change is large enough, the income of unskilled workers is high and will converge to a higher level in the next period, which means that the skill-biased technological change makes the distribution of wealth follow an egalitarian transition. In Figure 6, this change is indicated by the black arrows. This egalitarian transition then decreases the skill premium in the long-run, as indicated by the white arrow. Therefore, a large enough skill-biased technological change generates a Kuznets curve in the skill premium: it increases the skill premium when it happens but also makes the distribution of wealth follow an egalitarian transition; when it stops, the egalitarian transition decreases the skill premium, as indicated by the white arrows.

We can offer some explanations for the non-monotonic behaviour of the skill premium of the U.S. in the 20th century. The technological change at the beginning of the 20th century was unskill-biased. Mass production and assembly lines replaced skilled workers and broke down the production process into a series of elementary tasks that could be performed by unskilled workers. This change encouraged firms to increase productivity for unskilled workers, which increased the unskilled wage. According to the previous analysis, both the direct and the indirect effects of unskill-biased technological change decrease the skill premium. As a result, the skill premium kept decreasing in the first half of the 20th century.

The technological change has been skill-biased since the midpoint of the century. The new technology, computers and automatons for example, replaced unskilled workers and encouraged firms to increase productivity for skilled workers. Autor, Katz and Krueger (1998) suggest that the growth of relative demand for skilled workers, which could be largely explained by the spread of computer technology, was still rapid in 1995, which implies that the skill-biased technological change was still in progress at the end of last century. According to the previous analysis, the direct effect of sustaining skill-biased technological change could explain the increase in the skill premium in the second half of the 20th century. Moreover, we can predict that if the current skill-biased technological change is large enough, when it stops, the skill premium will decrease since the distribution of wealth will follow an egalitarian transition.

### *4.3 Financial Aid in the 1960s-1970s*

The skill-biased technological change was accelerated in the 1970s, according to Autor, Katz and Krueger (1998). However, there was a decline in the skill premium in the 1970s. A possible reason for this decline is the increased financial aid for college from the government. Government financial aid for higher education increased by a large amount in the U.S. from the late 1960s to the early 1970s (Mcpherson and Schapiro (1991)). This change reduced the imperfection of the credit market and decreased the gap between saving and borrowing interest rate. As a result, in the short-run the skill premium was decreased, according to Proposition 5. Furthermore, according to Proposition 8, the reduction in the borrowing interest rate can also cause the transition of the distribution of wealth to follow to an egalitarian one, which decreases the skill premium further. Hence the skill premium kept decreasing in the 1970s. Until the effect of the accelerated skill-biased technological change started to dominate, the skill premium started to increase again.

## 5 Conclusion

In order to analyze the patterns of the skill premium, this paper builds an OLG model, looking into a labour market with endogenous supply and endogenous demand of different types of workers. The paper discusses not only the short-run effects of a series of determinants on the skill premium, but also their long-run effects with dynamic distribution of wealth. Major results include:

The static equilibrium of the model shows that the difference of the skill premium across countries is caused by a variety of reasons, such as the cost of education, the imperfection of the credit market, distribution of wealth, and access to technology. The higher cost of education, the more severe credit market imperfection, the lower average wealth, and more skill-biased technological access lead to a larger skill premium.

In the long-run, the model has two possible cases of steady state: the egalitarian one, where everyone has the same income and the polarized one, where the skill premium is large and the income is unequal. Since the transition of the distribution of wealth interacts with the skill premium, exogenous shocks can affect the skill premium in the long-run by affecting the transition of the distribution of wealth and making it lead to a different steady state. The effects of technological changes could explain the non-monotonic behaviour of the skill premium of the U.S. in the 20th century.

# Appendix

## ***Proof of Lemma 2***

Let labour ratio $l_t = \frac{L_{s,t}}{L_{u,t}}$, and we can rewrite productivities as functions of labour ratio:

$$A_{u,t} = \left( \frac{B/\gamma}{1 + (\gamma/\delta)^{\sigma/(\omega-\sigma)} l_t^{\omega\sigma/(\omega-\sigma)}} \right)^{\frac{1}{\omega}} \tag{21}$$

$$A_{s,t} = \left( \frac{B/\delta}{1 + (\gamma/\delta)^{\sigma/(\sigma-\omega)} l_t^{\omega\sigma/(\sigma-\omega)}} \right)^{\frac{1}{\omega}} \tag{22}$$

Let $\mathcal{L}_t = (\gamma/\delta)^{\sigma/(\omega-\sigma)} l_t^{\omega\sigma/(\omega-\sigma)}$, and plug (21) and (22) into the first order conditions for problem (12), then we can show wages as functions of labour ratio:

$$w_{u,t} = \left(\frac{r}{\alpha}\right)^{\frac{\alpha}{1-\alpha}} (1-\alpha)(1+\mathcal{L}_t)^{\frac{1-\sigma}{\sigma}} (\frac{B/\gamma}{1+\mathcal{L}_t})^{\frac{1}{\omega}} \tag{23}$$

$$w_{s,t} = \left(\frac{r}{\alpha}\right)^{\frac{\alpha}{1-\alpha}} (1-\alpha)(1+\frac{1}{\mathcal{L}_t})^{\frac{1-\sigma}{\sigma}} (\frac{B/\delta}{1+\frac{1}{\mathcal{L}_t}})^{\frac{1}{\omega}} \tag{24}$$

then we have

$$\frac{dw_{u,t}}{dl_t} > 0 \tag{25}$$

$$\frac{dw_{s,t}}{dl_t} < 0 \tag{26}$$

and

$$\lim_{l_t \to +0} w_{s,t} = +\infty \tag{27}$$

$$\lim_{l_t \to +\infty} w_{u,t} = +\infty \tag{28}$$

## ***Proof of Lemma 3***

Let

$$G(l_t) = w_{s,t} - h(1+r) - w_{u,t}(1+r)$$

From (25), (26), (27), and (28) we have

$$\lim_{l_t \to +0} G(l_t) = +\infty$$

$$\lim_{l_t \to +\infty} G(l_t) = -\infty$$

$$\frac{dG(l_t)}{dl_t} < 0$$

Therefore, only one unique $(w_{u,t}^0, w_{s,t}^0)$ solves $w_{s,t} - h(1+r) = w_{u,t}(1+r)$, and condition (6) only holds when $\frac{w_{s,t}}{w_{u,t}} \geq \frac{w_{s,t}^0}{w_{u,t}^0}$.

## ***Proof of Proposition 5***

Because $L_{u,t}^S = (f_t - M + \varepsilon)/2\varepsilon$ and $L_{s,t}^S = (M + \varepsilon - f_t)/2\varepsilon$, we have:

$$l_t = \frac{(M + \varepsilon - f_t)/2\varepsilon}{(f_t - M + \varepsilon)/2\varepsilon} \tag{29}$$

We can solve for $f_t$:

$$f_t = \frac{M + \varepsilon + l_t(M - \varepsilon)}{l_t + 1} \tag{30}$$

By replacing the $f_t$, $w_{u,t}$ and $w_{s,t}$ in (7) with (23) (24) and (30), we have the reduced equation:

$$\begin{aligned} & \frac{M+\varepsilon+l_t(M-\varepsilon)}{l_t+1}(i-r) \\ = \; & h(1+i) \\ & +\left(\frac{r}{\alpha}\right)^{\frac{\alpha}{1-\alpha}}(1-\alpha)\left[(1+r)(\mathcal{B}/\gamma)^{\frac{1}{\omega}}(1+\mathcal{L}_t)^{\frac{1-\sigma}{\sigma}-\frac{1}{\omega}}-(\mathcal{B}/\delta)^{\frac{1}{\omega}}(1+\frac{1}{\mathcal{L}_t})^{\frac{1-\sigma}{\sigma}-\frac{1}{\omega}}\right] \end{aligned}$$

Let

$$\begin{aligned} F(l_t) \; = \; & \frac{M+\varepsilon+l_t(M-\varepsilon)}{l_t+1}(i-r)-h(1+i)- \\ & \left(\frac{r}{\alpha}\right)^{\frac{\alpha}{1-\alpha}}(1-\alpha)\left[(1+r)(\mathcal{B}/\gamma)^{\frac{1}{\omega}}(1+\mathcal{L}_t)^{\frac{1-\sigma}{\sigma}-\frac{1}{\omega}}-(\mathcal{B}/\delta)^{\frac{1}{\omega}}(1+\frac{1}{\mathcal{L}_t})^{\frac{1-\sigma}{\sigma}-\frac{1}{\omega}}\right] \\ = \; & 0 \end{aligned} \tag{31}$$

The $L^*_{u,t}$ and $L^*_{s,t}$ solving (31) are the equilibrium proportions of workers.

For $l_t \in (0, +\infty)$, $\mathcal{L}_t = (\gamma/\delta)^{\sigma/(\omega-\sigma)} l_t^{\omega\sigma/(\omega-\sigma)}$ yields:

$$\lim_{l_t \to +0} \mathcal{L}_t = 0 \tag{32}$$

$$\lim_{l_t \to +\infty} \mathcal{L}_t = +\infty \tag{33}$$

$$\frac{d\mathcal{L}_t}{dl_t} = (\gamma)^{\sigma/(\omega-\sigma)} \omega\sigma/(\omega-\sigma) l_t^{\omega\sigma/(\omega-\sigma)-1} > 0 \tag{34}$$

Rewriting the assumption $\omega > \sigma/(1-\sigma)$ yields:

$$\frac{1-\sigma}{\sigma} - \frac{1}{\omega} > 0 \tag{35}$$

Then we have:

$$\lim_{l_t \to +0} F(l_t) = (M+\varepsilon)(i-r)-h(1+i)-\left(\frac{r}{\alpha}\right)^{\frac{\alpha}{1-\alpha}}(1-\alpha)\left[-(\mathcal{B}/\delta)^{\frac{1}{\omega}}(1+\frac{1}{\lim\limits_{l_t \to +0}\mathcal{L}_t})^{\frac{1-\sigma}{\sigma}-\frac{1}{\omega}}\right] = +\infty \tag{36}$$

$$\lim_{l_t \to +\infty} F(l_t) = (M-\varepsilon)(i-r)-h(1+i)-\left(\frac{r}{\alpha}\right)^{\frac{\alpha}{1-\alpha}}(1-\alpha)\left[(1+r)(\mathcal{B}/\gamma)^{\frac{1}{\omega}}(1+\lim_{l_t \to +\infty}\mathcal{L}_t)^{\frac{1-\sigma}{\sigma}-\frac{1}{\omega}}\right] = -\infty \tag{37}$$

$$\begin{aligned} F_{l_t} &= \frac{-2\varepsilon(i-r)}{(l_t+1)^2} - \left(\frac{r}{\alpha}\right)^{\frac{\alpha}{1-\alpha}}(1-\alpha)\left(\frac{1-\sigma}{\sigma}-\frac{1}{\omega}\right)\cdot \\ & \quad \left[(1+r)(\mathcal{B}/\gamma)^{\frac{1}{\omega}}(1+\mathcal{L}_t)^{\frac{1-\sigma}{\sigma}-\frac{1}{\omega}-1} + (\mathcal{B}/\delta)^{\frac{1}{\omega}}(1+\frac{1}{\mathcal{L}_t})^{\frac{1-\sigma}{\sigma}-\frac{1}{\omega}-1}\cdot\left(\frac{1}{\mathcal{L}_t}\right)^{-2}\right]\cdot\frac{d\mathcal{L}_t}{dl_t} \\ &< 0 \end{aligned} \tag{38}$$

Therefore function $F(l_t)$ has one and only one root in $(0, +\infty)$.

We regard $h, \beta(= i-r), M$ and $\gamma$ as variables and consider the partial derivatives of $F$ with respect to them:

$$F_h = -(1+i) < 0 \tag{39}$$

$$F_M = i - r > 0 \tag{40}$$

$$F_\gamma = \left(\frac{r}{\alpha}\right)^{\frac{\alpha}{1-\alpha}}(1-\alpha)\left[(1+r)\mathcal{B}^{\frac{1}{\omega}}\gamma^{-\frac{1}{\omega}-1}(1+\mathcal{L}_t)^{\frac{1-\sigma}{\sigma}-\frac{1}{\omega}}\right]\frac{1}{\omega} > 0 \tag{41}$$

$$F_\delta = -\left(\frac{r}{\alpha}\right)^{\frac{\alpha}{1-\alpha}}(1-\alpha)\left[\mathcal{B}^{\frac{1}{\omega}}\delta^{-\frac{1}{\omega}-1}(1+\frac{1}{\mathcal{L}_t})^{\frac{1-\sigma}{\sigma}-\frac{1}{\omega}}\right]\frac{1}{\omega} < 0$$

$$F_\beta = \left[\frac{M+\varepsilon+l_t(M-\varepsilon)}{l_t+1} - h\right](1+r) \tag{42}$$

According to condition (6) we have

$$w_{s,t} - w_{u,t}(1+r) > h(1+r)$$

By multiplying both sides with -1 and adding $h(1+i)$ to both sides we have

$$w_{u,t}(1+r) - w_{s,t} + h(1+i) < h(i-r)$$

which implies

$$\begin{aligned} F_{\beta} &= \left[\frac{M+\varepsilon+l_t(M-\varepsilon)}{l_t+1} - h\right](1+r) \\ &= \left\{\frac{1}{i-r}[w_{u,t}(1+r)+h(1+i)-w_{s,t}] - h\right\}(1+r) \\ &< 0 \end{aligned}$$

Then we have $\frac{dl_t}{dh} < 0$, $\frac{dl_t}{dM} > 0$, $\frac{dl_t}{d\gamma} > 0$, $\frac{dl_t}{d\delta} < 0$ and $\frac{dl_t}{d\mathcal{B}} < 0$, which imply the effects of the parameters on the labour ratio. In addition, the effects of the parameters on the skill premium can be derived according to Equation (13).

## *Proof of Proposition 7*

If $(1-\theta)(1+i) < 1$,

There are only three possible values for the bequests in the steady state: $\overline{x}_{\text{I}}, \overline{x}_{\text{II}}$ and $\overline{x}_{\text{III}}$,because any bequest that is not equal to any one of them will converge to one of them.

$\overline{x}_{\text{I}}$ and $\overline{x}_{\text{II}}$ exist in the steady state at the same time require $\overline{x}_{\text{I}} < f < \overline{x}_{\text{II}}$. $\overline{x}_{\text{I}} < f$ implies that

$$w_u > \frac{\theta+\theta r-r}{(\theta+\theta i-i)(1+r)}\left[w_s - h(1+i)\right] \tag{43}$$

while $f \leq \overline{x}_{\text{II}}$ implies that

$$w_u < \frac{\theta + \theta r - r}{(\theta + \theta i - i)(1 + r)} [w_s - h(1 + i)] \tag{44}$$

There is a contradiction between (43) and (44), thus $\overline{x}_{\text{I}}$ and $\overline{x}_{\text{II}}$ cannot exist in the steady state at the same time.

$\overline{x}_{\text{II}}$ and $\overline{x}_{\text{III}}$ exist in the steady state at the same time require $\overline{x}_{\text{II}} < h < \overline{x}_{\text{III}}$. $h < \overline{x}_{\text{III}}$ implies that

$$(1 - \theta) w_s > h \tag{45}$$

$\overline{x}_{\text{II}} < h$ implies that

$$(1 - \theta) w_s < h \tag{46}$$

There is contraction between (45) and (46), thus $\overline{x}_{\text{II}}$ and $\overline{x}_{\text{III}}$ cannot exist in the steady state at the same time.

$\overline{x}_{\text{I}}$ and $\overline{x}_{\text{III}}$ exist in the steady state at the same time require $\overline{x}_{\text{I}} < f < h < \overline{x}_{\text{III}}$. From the previous analysis, $h < \overline{x}_{\text{III}}$ implies that $h < \overline{x}_{\text{II}}$; while $\overline{x}_{\text{I}} < f$ implies that $\overline{x}_{\text{II}} < f$. Then there must be $f > h$, which violates $f < h$. Thus $\overline{x}_{\text{I}}$ and $\overline{x}_{\text{III}}$ cannot exist in the steady state at the same time.

Therefore, there could be only one level of bequest in the steady state, which means that the steady state is always an egalitarian one.

If $(1 - \theta)(1 + i) > 1$,

In this situation, $\overline{x}_{\text{II}}$ is not a stable point any more. So $\overline{x}_{\text{I}}$ and $\overline{x}_{\text{III}}$ exist in the steady state at the same time require $\overline{x}_{\text{I}} < f < h < \overline{x}_{\text{III}}$, which implies

$$w_u < \frac{\theta + \theta r - r}{(\theta + \theta i - i)(1 + r)} [w_s - h(1 + i)] \tag{47}$$

$$(1-\theta)w_s > h \tag{48}$$

When these two conditions hold together, it is a polarized steady state, while if they are violated, it is an egalitarian one.

## *Proof of Proposition 9*

According to the proof of Proposition 7, the condition for $\overline{x}_{\mathrm{I}} \leq f \leq \overline{x}_{\mathrm{II}}$ to hold is

$$w_{u,t} \leq \frac{\theta+\theta r - r}{(\theta+\theta i - i)(1+r)}\left[w_{s,t} - h(1+i)\right] \tag{49}$$

By rewriting it we have:

$$w_{u,t} + \frac{\theta+\theta r - r}{(i-\theta-\theta i)(1+r)} w_{s,t} \leq \frac{\theta+\theta r - r}{(i-\theta-\theta i)(1+r)} h(1+i) \tag{50}$$

According to Proposition 5 and Equation (24), the skilled wage $w_{s,t}$ increases when $\gamma$ decreases. Also, skill premium $\frac{w_{s,t}}{w_{u,t}}$ decreases when $\gamma$ decreases, which implies that $w_{u,t}$ must increase when $\gamma$ decreases. Since $\frac{\theta+\theta r-r}{(i-\theta-\theta i)(1+r)} > 0$, the left-hand side of Equation (50) increases when $\gamma$ decreases and the left-hand side of Equation (50) is infinite when $\gamma \to 0$. Therefore, when $\gamma$ is small enough, Equation (49) is violated and the model reaches an egalitarian steady state. A similar conclusion can be proved for parameter $\delta$.

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